

Mediterranean Youth Mathematical Championship (MYMC)
Trieste, July 8, 2015

Morning round

WE1

(Leonardo Pisano, *Liber Abaci*, 1202)

There were two men: the first had 12 fish, while the second had 13. The fish all were equally priced. From the first man the customs agent took away 1 fish and 12 denari. From the second man the customs agent took 2 fish, and gave the man back 7 denari. Find the price of each fish in denari.

- A) Less than 22
- B) 22
- C) Between 22 and 23
- D) 23
- E) More than 23

WE2

Two angles of the same measure α are shown in the figure; therefore, the angle marked "?" is equal to:

- A) α
- B) 2α
- C) 3α
- D) 4α
- E) 5α



WE3

(Leonardo Pisano, *Liber Abaci*, 1202)

There is a vat which has four different holes. The vat empties at a different rate depending on which hole is opened: through the first hole it takes 1 day; through the second it takes 2 days; through the third it takes 3 days; and through the fourth it takes 4 days. If all four holes are opened at the same time, how many hours will it take for the vat to empty?

- A) More than 13 hours
- B) 13 hours
- C) Between 12 and 13 hours
- D) 12 hours
- E) Less than 12 hours

WE4

The product of two positive numbers x and y is 3 times their sum; the same product is equal to 6 times the difference between the two numbers. We assume $x \geq y$. What are the values of x and y ?

WE5

Joseph had his 31st birthday on the 1st January 2015, and he noticed that 2015 is a multiple of 31. Supposing that Joseph lives until he is 100 years old, how many other times will his age be a divisor of the year?

3

WE6

Let Q be a square with two of its vertices on one face of a cube, and its other two vertices on the opposite face of the cube. The cube has edges of length 1. What is the maximum value of the square of the perimeter of Q ? [We ask for the value of $(\text{perimeter of } Q)^2$.]

18

WE7

The 24 inhabitants of a village on a distant island are of two kinds: the knaves, who always lie, and the knights, who always tell the truth.

The first inhabitant says: 'In the village, the number of knaves is a multiple of 1.'

The second inhabitant says: 'In the village, the number of knaves is a multiple of 2.'

The third inhabitant says: 'In the village, the number of knaves is a multiple of 3.'

And so on until the final inhabitant, who says: 'In the village, the number of knaves is a multiple of 24.'

(We note that the inhabitants do not consider the number 0 as a multiple of other numbers, while every positive number is considered a multiple of itself.)

How many of the inhabitants are knaves?

WE8

A figure is composed of two right-angled triangles which share two vertices, such that the hypotenuse of the first triangle is a cathetus of the second (and the triangles have no other points in common). The triangles are similar and have integer sides.

What is the area of the smallest figure which satisfies these conditions?

WE9

An urn contains 7 balls, 3 of which are white and 4 of which are black.

All of the balls are taken out of the urn, one at a time and at random, and are placed in a row in the order in which they are removed.

What is the probability that the first two white balls in the row are consecutive?

- A) $1/4$
- B) $1/3$
- C) $3/7$
- D) $1/2$
- E) $4/7$

WE10

Let $A = \{490, 497, 506, 512\}$.

Determine how many of the elements of A divide the number $N = 46^6 - 25^6 + 39^6 - 32^6$.

- A) 0
- B) 1
- C) 2
- D) 3
- E) 4

6

$$9^6(9^{36} - 2^{36})$$
$$39^6 - 25^6$$
$$(39-25)(39+25)$$
$$14 \quad 64$$
$$2^7$$